1. **Circuit Equivalence**

Consider the circuits in Fig. 1 and answer the following questions:

(a) (3 points) Calculate the equivalent resistance as seen from A and B, A and C, and B and C.

(b) (3 points) Calculate the equivalent resistance as seen from D and E, D and F, and E and F.

(c) (3 points) Calculate a formula for S as a function of R so that both circuits are completely equivalent.

(d) (1 point) Explain what would change in the formula you obtained in part (c) if the resistances R and S were replaced by impedances in the s-domain.
2. LED Driver

A student needs to drive four LEDs using the digital output of a microcontroller. Each LED is rated at 1.2 V and 30 mA. Because the microcontroller output pin can output a maximum of 20 mA, the student needs to construct the circuit in Fig. 2 to drive the required $4 \times 30$ mA. Answer the following questions to help him or her select appropriate values for the resistors $R_B$ and $R_C$.

(a) (4 points) Assume that the BJT transistor is saturated and calculate the value of $R_C$ so that each LED has a voltage drop of 1.2 V with a current of 30 mA. Calculate the corresponding value of $v_2$ and the power absorbed by the resistor $R_C$.

(b) (4 points) Let $v_\gamma = 0.6$ V, a transistor current gain $\beta = 100$, and calculate the value of $R_B$ so that when $v_1 = 5$ V and each LED is operating at their rated voltage and current the transistor is saturated. Calculate the current through $R_B$ and the power it absorbs.

(c) (2 points) Explain how the circuit you helped design works as $v_1$ takes the values $v_1 = 0$ V and $v_1 = 5$ V. Does it keep the microcontroller output pin current below 20 mA?

3. OpAmp Design

In this question you will design an opAmp circuit to realize the transfer-function:

$$T(s) = \frac{10(s + 1,000)}{s^2 + 10,010s + 100,000}$$

(a) (3 points) Calculate positive real numbers $\alpha$ and $\beta$ so that

$$T(s) = \frac{\alpha}{s+10} + \frac{\beta}{s+10,000}$$

(b) (7 points) Design a circuit using opAmps that implements $T(s)$ as the sum of the two transfer-functions from part (a).
4. Circuit Analysis

Let $v_x(0) = 1$ V and answer the following questions:

(a) (4 points) Convert the circuit in Fig. 3 to the $s$-domain and formulate its node-voltage equations. Use the node-voltage and labels provided in the figure and clearly indicate the final equations and circuit variable unknowns. Make sure your final equations only involve node-voltages.

(b) (4 points) Convert the circuit in Fig. 3 to the $s$-domain and formulate its mesh-current equations. Use the mesh-currents and labels provided in the figure and clearly indicate the final equations and circuit variable unknowns. Make sure your final equations only involve mesh-currents.

(c) (2 points) Write $s$-domain expressions for the current through the voltage source $v_1$ and the voltage across the current source $i_2$ in terms of the node voltages and mesh currents from parts (a) and (b).
5. **OpAmp Circuit Analysis**

Assume zero initial-conditions for the circuits in Fig. 4 and answer the questions:

(a) (7 points) Convert the three circuits into the $s$-domain and show that they have identical transfer-functions

\[
\frac{V_o(s)}{V_i(s)} = \frac{-10s}{s + \frac{1}{RC}}.
\]

(b) (3 points) Let $v_i(t) = V_s u(t)$ and calculate the response $v_o(t)$.

6. **Frequency Response**

Consider the circuits in Fig. 4 and answer the following questions:

(a) (3 points) Calculate the magnitude and phase of the circuits’ frequency response at $\omega = 0$, $\omega = (RC)^{-1}$, and $\omega \to \infty$.

(b) (4 points) Sketch the magnitude and phase of the circuits’ frequency response. What kind of filter is the circuit?

(c) (3 points) Calculate the steady-state response $v_o^{ss}(t)$ produced in response to an input voltage $v_i(t) = 1 + \cos((RC)^{-1}t)$.

(d) (2 points (bonus)) Which of the circuits has a problem with saturation? Explain.

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Have a great Winter break!