1. **Gain Analysis**

Connect $v_C$ and $v_D$ to the ground in the circuit in Fig. 1 and answer the following questions:

(a) (3 points) Calculate the gains: $v_i$ to $v_A$, $v_A$ to $v_B$, $v_B$ to $v_o$.

**Solution:**

Because the first opAmp is a standard inverting opAmp circuit:

$$\frac{v_A}{v_i} = -\frac{10R_1}{R_1} = -10. \quad (+1 \text{ point})$$

Because second opAmp is a standard inverting opAmp circuit:

$$\frac{v_B}{v_A} = -\frac{10R_2}{R_2} = -10. \quad (+1 \text{ point})$$

From voltage division

$$\frac{v_o}{v_B} = \frac{R_3}{R_3 + R_3} = \frac{1}{2}. \quad (+1 \text{ point})$$

(b) (1 point) Calculate the gain: $v_i$ to $v_o$.

**Solution:**

Because there is no loading of any circuit

$$\frac{v_o}{v_i} = \frac{v_o}{v_B} \frac{v_B}{v_A} \frac{v_A}{v_i} = (-10)(-10)rac{1}{2} = 50 \quad (+1 \text{ point})$$
(c) (4 points) Calculate the gain $v_s$ to $v_o$ if a circuit with Thevenin voltage $v_s$ and equivalent resistance $2R_1/3$ is connected to the input $v_i$.

**Solution:**

It is enough to analyze the first stage:

![Circuit Diagram]

Replace the source, $2R_1/3$ and $2R_1$ by its Thevenin equivalent:

![Circuit Diagram]

where

$$v_T = \frac{2R_1}{2R_1 + \frac{2}{3}R_1} v_s = \frac{3}{4} v_s, \quad R = \frac{1}{\frac{2R_1}{3} + \frac{3}{2R_1}} = \frac{R_1}{2}$$

so that

$$v_A = -\frac{10R_1}{\frac{R_1}{2} + R_1} v_T = -\frac{20}{3} \frac{3}{4} v_s = -5v_s$$

which has half the gain of the original circuit so that $v_o/v_i = 50/2 = 25$. (+1 point)

(d) (1 point) If a circuit is to be connected to $v_i$ as in part (c), show how you could add an opAmp circuit before connecting the circuit to $v_i$ to make the gain from $v_s$ to $v_o$ equal to the gain from $v_i$ to $v_o$.

**Solution:**

One could use a buffer:

![Circuit Diagram]

2. Voltage Offset
Let \( v_C = v_D = 1 \text{V} \) and answer the following questions regarding the opAmp circuit in Fig. 1:

(a) (3 points) Show that \( v_A = 11 - 10v_i, \) \( v_B = 100v_i - 99, \) \( v_o = 50v_i - 49.5. \)

**Solution:** Calculate using the subtractor formulas or linearity:

\[
\begin{align*}
v_A &= \frac{11R}{R}v_C - \frac{10R}{R}v_i = 11 - 10v_i, \quad (+1 \text{ point}) \\
v_B &= \frac{11R}{R}v_D - \frac{10R}{R}v_A = 100v_i - 99, \quad (+1 \text{ point}) \\
v_o &= \frac{R}{2R}v_B = 50v_i - \frac{99}{2} \quad (+1 \text{ point})
\end{align*}
\]

(b) (3 points) Let the signal \( v_i = 1 + v_r \) in which \(-0.01 \text{V} \leq v_r \leq 0.01 \text{V}\). Calculate the corresponding range for the signal \( v_o \) and the the gain from \( v_r \) to \( v_o \).

**Solution:**

Calculate

\[
v_o = 50(1 + v_r) - 49.5 = 50v_r + 0.5 \quad (+1 \text{ point})
\]

from which the gain is 50 and

\[
0 \leq v_o = 50(1 + v_r) - 49.5 = 50v_r + 0.5 \leq 1 \quad (+1 \text{ point})
\]

(c) (2 points) Calculate the largest possible value of \(-v_{CC}\) and the smallest possible value of \(+v_{CC}\) so that the voltages \( v_A \) and \( v_B \) do not saturate for a signal \( v_i = 1 + v_r \) as in part (b).

**Solution:**

Because \(-0.01 \leq v_r \leq 0.01\)

\[
0.9 \leq v_A = 11 - 10(1 + v_r) = 1 - 10v_r \leq 1.1
\]

and \( v_A \) does not saturate if \(+v_{CC} \geq 1.1 \) and \(-v_{CC} \leq 0.9. \quad (+1/2 \text{ point})

Likewise

\[
0 \leq v_B = 100(1 + v_r) - 99 = 1 + 100v_r \leq 2
\]

and \( v_B \) not to saturate if \(+v_{CC} \geq 2 \) and \(-v_{CC} \leq 0. \quad (+1/2 \text{ point})

Therefore \(+v_{CC} = 2\) and \(-v_{CC} = 0\) are the largest and smallest possible. \quad (+1 \text{ point})

(d) (1 point) Let \(+v_{CC} = 2\text{V}\) and \(-v_{CC} = 0\text{V}\) and design an additional circuit with access only to theses voltages that when connected to the circuit in Fig. 1 is such that \( v_C = v_D = 1\text{V}. \)

**Solution:**

A simple voltage-divider with equal resistors suffices:
3. **OpAmp Design**

Design an opAmp circuit that implements the following operations \( v_o = 2(v_1 - v_2) + v_3 \)

(a) (3 points) Using any number of opAmps.

(b) (3 points) Using at most two opAmps.  

(c) (3 points) Using at most one opAmp.

**Solution:**

Here is one solution with 1 opAmp:

Using linearity

\[
v_o = -2v_2 + \frac{2}{3}v_1 + \frac{1}{3}v_3 = 2(v_1 - v_2) + v_3.
\]

NOTE TO GRADERS: Any correct solution with more than 2 opAmps earns 3 points.

NOTE TO GRADERS: Any correct solution with 2 opAmps earns 6 points.

NOTE TO GRADERS: Any correct solution with 1 opAmp earns 9 points.

NOTE TO GRADERS: If a solution is correct but uses inverted sources or signals, discount 1 point per inverted source.