1. Is the system
\[
\dot{x}(t) = A(t)x(t) + B(t)u(t), \quad x(t_0) = x_0
\]
\[
y(t) = C(t)x(t) + D(t)u(t), \quad t \geq t_0
\]
(1)
linear? Do you need any assumptions on the model, i.e. the \(A(t), B(t), C(t), D(t)\) matrices, or otherwise to reach your conclusion?

2. Can you solve the linear differential equation
\[
\dot{x}(t) = \frac{x(t)}{1-t}, \quad x(t_0) = x_0, \quad t \geq t_0
\]
What happens around \(t = 1\)?

3. Can you solve the non-linear differential equation
\[
\dot{y}(t) = \frac{\sin(2y(t))}{2(1-t)}, \quad y(t_0) = y_0, \quad t \geq t_0
\]
What happens around \(t = 1\)?

4. Show that if \(x(t)\) and \(y(t)\) are such that
\[
\dot{x}(t) = \frac{x(t)}{1-t}, \quad x(t_0) = x_0,
\]
\[
y(t) = \arctan(x(t)), \quad t \geq t_0
\]
then \(x(t)\) and \(y(t)\) solve the differential equations in problems 2. and 3. Is this system linear? Is it time-invariant?

5. Consider systems that process the input as follows:
\[
L_1(u(t)) = \int_{t_0}^{t} u(\tau) \, d\tau
\]
\[
L_2(u(t)) = \frac{1}{t-t_0} \int_{t_0}^{t} u(\tau) \, d\tau
\]
\[
L_3(u(t)) = \frac{1}{\delta} \int_{t-\delta}^{t} u(\tau) \, d\tau, \quad \delta > 0
\]
where \(t \geq t_0\). For each system \(L_i, i = 1, 2, 3\), answer the following questions:
(a) Is the system linear?
(b) Is the system time-invariant?
(c) What does the system do to the input \(u(t)\)?
(d) Find a representation in state-space form (1).
(e) What is the dimension of the state?
(f) What is the impact of the initial state in the solution?